1

(1)　次の定積分を求めよ．ただし，m と n は正の整数とする．
(Find the following definite integral. Here, m and n are positive integers.)

a) $\displaystyle\int_{-\pi}^{\pi} \sin mx \cdot \cos nx \, dx$

b) $\displaystyle\int_{-\pi}^{\pi} \sin mx \cdot \sin nx \, dx$

c) $\displaystyle\int_{-\pi}^{\pi} \cos mx \cdot \cos nx \, dx$

(2)　次の関数の極限を求めよ．
(Find the limit of the following function.)

$$\lim_{x\to 0}\left(\frac{6\cos x + 3x^2 - 6}{x^4} - \frac{\tan x - x}{2x^3}\right)$$

(3)　次の式で示す曲線 A はレムニスケート（連珠形）と呼ばれる．この曲線で囲まれる図形の面積 S を求めよ．
(Curve A expressed by the following equation is called as lemniscate. Find the area S of the shape enclosed by curve A.)

$$(x^2 + y^2)^2 - 2a^2(x^2 - y^2) = 0 \quad (a > 0)$$

2

(1)　つぎの行列 $\mathbf{A}$ について考える．
(Consider the following matrix $\mathbf{A}$.)

$$\mathbf{A} = \begin{bmatrix} -1 & -1 & 1 & -1 \\ -2 & 0 & 2 & -1 \\ 0 & -1 & 2 & 0 \\ 3 & 0 & 3 & 2 \end{bmatrix}$$

逆行列 $\mathbf{A}^{-1}$ を求めよ．
(Find the inverse matrix $\mathbf{A}^{-1}$.)

(2)　つぎの行列 $\mathbf{A}$ について考える．
(Consider the following matrix $\mathbf{A}$.)

$$\mathbf{A} = [\boldsymbol{a}_1 \quad \boldsymbol{a}_2 \quad \boldsymbol{a}_3] = \begin{bmatrix} 1 & a & 6 \\ 1 & -1 & 2 \\ 0 & 2 & -2 \end{bmatrix}$$

a)　$a = -4$ のとき，$\mathbf{A}$ の固有値を全て求めなさい．
(Find the all eigenvalues of matrix $\mathbf{A}$ when $a = -4$.)

b)　a)で求めた固有値 λ に対応する固有ベクトルをすべて求めなさい．　すべての固有ベクトルの x 成分は1とする．
(Find the all eigenvectors corresponding to the eigenvalues found in a) , when value of the all eigenvectors in x component is 1.)

c)　$a = 3$ のとき，3つのベクトル $\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{a}_3$ からグラム・シュミットの正規直交化法を用いて正規直交基底 $\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3$ を求めよ．
(Find orthonormal basis $\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3$ from the three vectors $\boldsymbol{a}_1$, $\boldsymbol{a}_2$, and $\boldsymbol{a}_3$ by means of the Gram-Schmidt orthonormalization when $a = 3$.)

3

以下の問いに答えなさい. ただし, y は独立変数 x の関数であり, その導関数を $y' = \frac{dy}{dx}$, $y'' = \frac{d^2y}{dx^2}$, $y''' = \frac{d^3y}{dx^3}$ と表す.

(Solve the problems below. y is a function of the independent variable x. The first, second and third derivatives are denoted by $y' = \frac{dy}{dx}$, $y'' = \frac{d^2y}{dx^2}$ and $y''' = \frac{d^3y}{dx^3}$ respectively.)

(1)　次の微分方程式について，一般解を求めなさい.
(Find the general solution of the following differential equations.)

a）　$y' + (1 + 2e^x)y = 0$

b)　$y'' + 2y' + 6x^2 = 0$

c）　$y''' + 2y'' = 0$

(2)　次の微分方程式について，初期条件 $y(0) = -4,\ y'(0) = 0$ を満たす特殊解を求めなさい.
(Find the particular solution of the following differential equation that satisfies the initial conditions $y(0) = -4,\ y'(0) = 0$.)

$$y'' + 4y' + 3y = -2e^{-2x}\sin x + 9x$$